

1. The first midterm (and the micro theory prelim) often include versions of the questions at the end of chapter 5 of MCWG. Questions on MCWG pages 160-166 that you should definitely know how to do are:

- 5.C.6
- 5.C.7
- 5.C.9
- 5.C.10
- 5.C.13 (try #6 below first)
- 5.D.5(a)

2. Find and characterize the relative extrema (critical points) of

a) $f(\mathbf{x}): \mathbb{R}^2 \rightarrow \mathbb{R}, f(\mathbf{x}) = x_1^2 - x_2^2$

b) $f(\mathbf{x}): \mathbb{R}^2 \rightarrow \mathbb{R}, f(\mathbf{x}) = x_1^4 + (x_1 - x_2)^4$

3. (UNR Theory Prelim, s2006)

For each production set Y : illustrate, explain what it means (in words), and provide an example:

- a) that does NOT have this property: $\mathbf{y} \in Y$ and $\mathbf{y} \geq 0 \rightarrow \mathbf{y} = 0$
- b) that has this property: $\mathbf{y} \in Y$ and $\mathbf{y} \neq 0 \rightarrow -\mathbf{y} \in Y$
- c) that has this property: $\forall \mathbf{y} \in Y$ and $\alpha \in [0, 1], \alpha \mathbf{y} \in Y$

4. (UNR Theory Prelim, s2006)

Given this claim written formally using MCWG notation, explain it (in words), then prove it:

$$Y = \{\mathbf{y}: (-\mathbf{z}, q), q \leq f(\mathbf{z}); \mathbf{z} \geq 0\} \text{ and } f(\mathbf{z}) \text{ homogeneous of } ^{\circ}1 \rightarrow \forall \mathbf{y} \in Y \text{ and } \alpha \geq 0, \alpha \mathbf{y} \in Y$$

5. (first day of class problem) Consider a producer problem of choosing the optimal mix of inputs (x_1, x_2) used to produce output q , where $q = A[x_1^a + x_2^a]^{1/a}$ where the output price is \$1.00 per unit, each input costs \$1.00/unit; $a = 1/2$, and the total expenditure on inputs cannot exceed \$360.00. Verify that your solution actually *maximizes* profit.

6. (Intermediate Micro question) If this product were free, 80 units would be demanded. And no-one is willing to pay more than 20\$ for it. Production costs are $C(q) = \$400 + \$3q + \$\frac{1}{4}q^2$.

- a) What is this establishment's *revenue-maximizing* level, q^M , of sales?
- b) What price will people pay for q^M ?
- c) What is the *total cost* of q^M ?
- d) What are *average cost* and *marginal cost* at q^M ?
- e) What is the *profit maximizing* level of sales?
- f) What can be learned from this question?

7. Should a government that cares about the happiness of all its variety-loving citizens intervene to stabilize the output price faced by producers at the average price between the lowest (p_L) and highest (p_H)? Do the consequences of such a strategy depend on whether or not the sector's technology displays non-decreasing returns to scale? Formally prove both answers.

8. (Princeton University, Dept. of Economics; General Exam For Doctor of Philosophy)

True or False, and PROVE: If the *production set* is closed and exhibits free disposal, then any feasible input mix \mathbf{z}^* that maximizes profit also minimizes cost for the same level of output.

9. (University of Minnesota, Economics Micro Theory Prelim) Prove the first two *Fundamental Theorems of Welfare Economics*.