

READ MCWG chapter 5 sections A-B-C

1. find the logical error on page 128 of MCWG
2. Consider a simple version of the Constant Elasticity of Substitution (“CES”) production function of two inputs  $(x_1, x_2)$  to produce output  $q$ , where  $q = A[\alpha_1 x_1^\rho + \alpha_2 x_2^\rho]^{1/\rho}$ 
  - a.
    - (i) Express the *marginal rate of technical substitution*.
    - (ii) Identify the degree of *homogeneity*.
  - b. using MCWG notation:
    - i) formally express the *netput vector*
    - ii) formally express the *production plan*. Explain the difference between the netput vector and the production plan, if there is any.
    - iii) formally express the *production set*
    - iv) verify, then NAME which of the following properties this *production set* displays:
      - (a)  $\mathbf{Y} \cap \mathbf{R}_+^2 \subset \{0\}$
      - (b)  $0 \in \mathbf{Y}$
      - (c)  $\mathbf{Y} - \mathbf{R}_+^2 \subset \mathbf{Y}$
      - (d)  $\forall \beta \geq 0, \beta \mathbf{y} \subset \mathbf{Y}$
    - v) formally express the *transformation function*
    - vi) formally express the *transformation frontier*
3. Assume the producer’s objective is to maximize profit. Given the CES function in (2), input prices denoted “ $w_1$ ” and “ $w_2$ ”, output price “ $p$ ,” and a limit of “ $M$ ” on the total expenditure on inputs, set up and solve the producer’s constrained optimization problem.
4.
  - a. Write out the *Jacobian* for problem 3.
  - b. Write out the *bordered Hessian* (I recommend the Chiang handout, but its also explained in MCWG appendix MD).
  - c. Restrict  $A > 0$  and  $\alpha_i > 0$ . Is the bordered Hessian ND, NSD, PD, or PSD, and, what restriction must be satisfied by the parameter  $\rho$  to ensure that your solution (3) is a maximum?

For Fun: Consider the *no free lunch* and *irreversibility* properties of some *production sets*. Can a *production set* satisfy the *no free lunch* property and be *reversible* at the same time? Consider MCWG problem 5.B.1. Find the solution to 5B1 in the “Microeconomic Theory by MCWG Solutions Manual” by Hara, Segal, and Tadelis. Is their answer correct? Why or why not? Draw a different *production set* that you think “violates irreversibility” (i.e., is reversible).