RECO 710 Fall 2009 Prof. Kilkenny Exercise 2 ~~due Wednesday, Sept 16,~~ 2009

GET INTO THE HABIT of repeating the question before you answer. GET INTO THE HABIT of formally stating all definitions and theorems you will use before you apply those definitions or theorems in your answer.

Comparative Statics is the analysis of the effects on optimal choices and outcomes arising from changes in the economic environment. There are two types of comparative static results. One is how the optimal value of the objective varies as a parameter changes. The other is how the optionally chosen instrument varies as a parameter changes. Practice finding comparative static results using the Implicit Function Theorem and the Envelope Theorem:

1. Given $q = f(x;\alpha) = \alpha + 8\alpha x - 2x^2$
where q is the objective
x is the instrument
α is a parameter
(a) find x^* that optimizes q
(b) write out the *value function*
(c) Use the Envelope Theorem to find $\frac{dq^*}{d\alpha}$. Verify your solution by solving for it another way.
(d) Use the Implicit Function Theorem to find $\frac{dx^*}{d\alpha}$. Verify by solving it some other way.

2. The *Cost Function* (a value function) for a computer chip producing firm that makes q chips denoted $C(q)$ exhibits increasing marginal costs. A fraction $(1-\alpha)$ of the chips it produces are defective and cannot be sold. The good chips sell for a price of p. The computer chip market is competitive.
(a) How do profits vary (increase, decrease, or no change) with respect to α? (HINT: Express the producer's profit max problem; then use the Envelope Theorem).
(b) How does the optimally chosen output level, q, vary with respect to α? (Use the ImpFTh).
(c) Suppose there are n identical computer chip producing firms in the market. Denote market demand by $D(p)$, and let $p(\alpha)$ be the competitive market price. How does the market price vary with α?

3. Consider the *production function* $y = f(\mathbf{x}) = Ax_1^{\alpha}x_2^{\beta}$
(a) Under what various conditions on α and β would $f(\mathbf{x})$ display:
(i) *decreasing returns to scale?*
(ii) *constant returns to scale?*
(iii) *increasing returns to scale?*
(b) **write** the *explicit function* for the *isoquant* at $\bar{y}$ for $f(\mathbf{x})$
(c) use the *Implicit Function Theorem* to solve for the *MRTS*
(d) How does the initial level of x_1 used affect the rate at which a producer would substitute x_1 for x_2 while still producing the same level of output?
(e) Let (w_1, w_2) denote the input prices, and C denote total cost: $C = w_1x_1 + w_2x_2$.
Using the method of *Lagrange* to) Find the (x_1^*, x_2^*) that minimizes total cost to produce $\bar{y}$.
(i) **illustrate** the solution graphically in $\mathbb{R}^2$ (with x_1 on the horizontal and x_2 on the vertical axis).
(ii) **express all** the FONCs

(iii) Are the FONCs homogenous explicit functions? (by *homogeneous function* we mean in the form F(**x**, λ; $\bar{y}$) = 0.)

(iv) Apply the Implicit Function Theorem to find (x_1*, x_2*) in the manner suggested by your illustration in (3.e.i).

4. (MCWG Exercise 5.C.2, page 138) Prove TWO WAYS that for any technology that has a production set Y that is closed and satisfies free disposal, the *profit function,* $\pi(p)$ is *convex.*