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Supply Theory sans Profit Maximization

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# Supply Theory sans Profit Maximization\*

Indraneel Dasgupta

## Abstract

We utilize the analytical construct of a stochastic supply function to provide an aggregate representation of a finite collection of standard deterministic supply functions. We introduce a consistency postulate for a stochastic supply function that may be satisfied even if no underlying deterministic supply function is rationalizable in terms of profit maximization. Our consistency postulate is nonetheless equivalent to a stochastic expansion of supply inequality, which summarizes the predictive content of the traditional theory of competitive supply. A number of key results in the deterministic theory follow as special cases from this equivalence. In particular, it yields a probabilistic version of the law of supply, which implies the traditional specification. Our analysis thus provides a necessary and sufficient axiomatic foundation for a de-coupling of the predictive content of the classical theory of competitive firm behavior from its a priori roots in profit maximization, while subsuming the traditional theory as a special case.

**KEYWORDS:** supply aggregation, stochastic supply function, stochastic consistency, weak axiom of profit maximization, stochastic supply inequality, law of supply

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## 1. Introduction

Suppose an industry contains 100 price-taking firms, which face identical input-output price configurations. Suppose further 50 of these firms choose the net output vector  $x$  under some price vector  $p$ , while the remaining 50 choose  $x'$  (say, because their technology is different). Then a natural way to model aggregate industry output is in terms of a *stochastic supply function* (SSF), which stipulates a choice of  $x$  and  $x'$ , each with probability  $\frac{1}{2}$ , under  $p$ . Analogously, when a competitive firm faces technology shocks governed by some exogenous stochastic process, its supply behavior can be modeled in terms of an SSF. Thus, for example, suppose the technology set available to some firm producing wheat is  $\tau$  if it snows at the beginning of the crop cycle, and  $\tau'$  otherwise. Given some price vector  $p$ , the firm chooses the net output vector  $x$  if its realized (available) technology is  $\tau$ ; it chooses  $x'$  otherwise. The probability of snowfall is  $\frac{1}{2}$ . Then the supply of this firm can be modeled in terms of the SSF outlined earlier. Both situations turn up routinely in applications of the theory of competitive firm behavior.<sup>1</sup>

What kind of minimal, intuitive *a priori* restriction would ensure that an SSF satisfies analogues of the empirical content of the traditional theory of competitive firm behavior? The classical theory of firm behavior posits that the output-input choices of a competitive firm, summarized by a *deterministic supply function* (DSF), satisfy the Weak Axiom of Profit Maximization (WAPM). WAPM implies the familiar Supply Inequality, which in turn yields the law of supply. For a firm's choices to be rationalizable in terms of profit-maximization with respect to some collection of feasible input-output combinations, it is necessary and sufficient that its DSF satisfy WAPM.<sup>2</sup> Thus, predictions regarding competitive supply behavior, whether at the firm level or at the industry level, are derived from WAPM. Once firms are permitted to violate WAPM, the classical theory fails to generate any empirical content whatsoever.

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<sup>1</sup> The first situation, for example, is commonly considered in applied studies of competitive industry behavior and efficiency measurement, stemming from the seminal theoretical work of Debreu (1951) and Farrell (1957). These contexts have given rise to a large microeconomic literature on stochastic production frontier estimation (see, for example, Kumbhakar and Lovell, 2000). In macroeconomics, the literature on real business cycles flowing from Kydland and Prescott (1982) routinely models firm technology as subject to stochastic shocks.

<sup>2</sup> See Section 2 for formal definitions. Samuelson (1947) introduced WAPM and the Supply Inequality, showed that the former implies the latter (and thereby, the law of supply), and noted that WAPM is necessary for profit maximization. See also Debreu (1959, p. 47). Hanoch and Rothschild (1972) showed that WAPM is sufficient for a firm's choices to be rationalizable in terms of profit-maximization with reference to some technology set. The name, WAPM, is due to Varian (1984).

Violations of WAPM appear to be frequently encountered in empirical studies. It has been argued that large departures from the profit maximization hypothesis, and, indeed, from maximizing behavior per se, are in fact routine in reality.<sup>3</sup>

In a well-known response, Becker (1962) argued that, even if individual firms do not maximize profits, the law of supply may hold in the aggregate as a market-wide phenomenon. An extension of his claim suggests that, in contexts with stochastic firm technology, the law of supply may also hold in the aggregate as a *firm-level* phenomenon even if the firm does not maximize profits with respect to its realized technology. Becker however based his case on examples. He did not provide general non-profit-maximizing behavioral foundations that would prove both necessary and sufficient for the empirical content of the traditional theory (as summarized by the Supply Inequality), to hold as an aggregate phenomenon. Recently, Dasgupta (2005) has posited a consistency restriction for a DSF which is weaker than WAPM, yet implies the Supply Inequality. However, Dasgupta left unexamined the connection between this restriction and aggregate supply behavior.

Regardless of whether an SSF provides an aggregate representation of the supply behavior of multiple price-taking firms with deterministic (but possibly different) technologies, or of a single competitive firm endowed with a stochastic technology, what kind of minimal, intuitively plausible, *a priori* consistency restriction can one posit for SSFs, that would imply aggregate supply behavior broadly in consonance with the classical (deterministic) theory? Would such a *necessary and sufficient* consistency restriction on the aggregate SSF representation be compatible with violation of WAPM by *every* underlying constituent DSF? Furthermore, would this restriction be compatible even with violation of the condition in Dasgupta (2005) by all underlying DSFs? If so, the empirical content of the classical theory (i.e. the Supply Inequality), suitably reinterpreted, would prove robust both: (i) as a market-wide phenomenon even when *no* individual firm exhibits choices that can be rationalized in terms of profit-maximization, and (ii) as a firm-level phenomenon even when the firm's choices on the basis of its realized technology set *invariably* violates profit-maximization. Such an analysis would provide a general axiomatic foundation for a complete de-linkage between the empirical content of the classical theory of competitive firm behavior and the assumption of profit maximization, while

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<sup>3</sup> See, for example, Leibenstein (1976, 1979, 1983) and Simon (1979). Varian (1985) provides a formal treatment of the notion of a 'large' violation in this context. Public sector firms are often actively prevented from maximizing profits, in pursuance of other public objectives, even though they are price-takers in some competitive markets and face politically determined prices in others. 'Firms' in our usage also includes foundations and other private non-profit organizations which are small enough to be reasonably thought of as price-takers.

subsuming both the analysis by Dasgupta and the traditional, WAPM-based theory, as special cases. Thus, it would identify the exact sense in which "... anti-marginalists can believe that firms are irrational, marginalists that market responses are rational, and both can be talking about the same economic world" (Becker, 1962: p.12).

This paper offers such an analysis. Furthermore, we show that the empirical content of the classical theory (i.e. a general formulation of the Supply Inequality) can be derived from an *a priori* behavioral restriction with independent intuitive appeal that does not presuppose even cost minimization.

Section 2 introduces the basic notation and definitions. In particular, we introduce our notion of a stochastic supply function and discuss how it can be used as an aggregate analytical representation of a finite class of deterministic supply functions. Section 3 introduces our *a priori* consistency postulate for SSFs, which we term *stochastic consistency*, and discusses its properties. We show that the aggregate SSF representation of a finite class of DSFs may satisfy stochastic consistency even if no underlying DSF can be rationalized in terms of profit maximization (nor satisfy cost minimization). Our consistency postulate may hold even if all underlying DSFs violate the consistency restriction proposed by Dasgupta. Thus, stochastic consistency completely delinks firm behavior from profit-maximization. We then introduce a general probabilistic expansion of the classical Supply Inequality, which is equivalent to our stochastic consistency. Existing results in the deterministic framework, such as the relationship between WAPM and the Supply Inequality and the equivalence between Dasgupta's condition and the Supply Inequality, turn out to be special cases of this general equivalence. Our general equivalence condition also yields a probabilistic version of the law of supply, from which the traditional version falls out as a special case. Section 4 concludes. Detailed proofs are presented in the Appendix.

## 2. Notation and preliminaries

Let  $n$  be the number of commodities and let  $N = \{1, 2, \dots, n\}$ . Let  $\mathfrak{R}$  and  $\mathfrak{R}_{++}$  denote, respectively, the set of real numbers and that of positive real numbers. A competitive firm faces  $n$ -dimensional vectors of commodity prices and produces  $n$ -tuples of net outputs. We shall denote price vectors by  $p, p'$  etc. and net output vectors by  $x, x'$  etc. The set of all possible price vectors is  $\mathfrak{R}_{++}^n$ . Given a net output vector  $x$ ,  $x_i$  will denote the amount of the  $i$ -th commodity contained in  $x$ . Given a price vector  $p$ ,  $p_i$  will denote the price of the  $i$ -th commodity. We shall denote the power set (i.e., the set of all possible subsets) of  $\mathfrak{R}^n$  by  $\pi(\mathfrak{R}^n)$ . Given any finite set  $\Omega$ ,  $|\Omega|$  will denote the number of elements in  $\Omega$ .

**Definition 2.1.** A *stochastic supply function* (SSF) is a rule  $s$ , which specifies, for every  $p \in \mathfrak{R}_{++}^n$ , exactly one finitely additive probability measure  $t$  on  $(\mathfrak{R}^n, \pi(\mathfrak{R}^n))$  ( $\mathfrak{R}^n$  being the set of outcomes and  $\pi(\mathfrak{R}^n)$  being the relevant algebra in  $\mathfrak{R}^n$ ).

Given an SSF,  $s$ , let  $t = s(p)$ , and let  $A$  be a subset of  $\mathfrak{R}^n$ . Then  $t(A)$  represents the probability a net output vector will be chosen from the set  $A$ , under the price vector  $p$ .

**Definition 2.2.** A *deterministic supply function* (DSF) is a rule  $S$ , which specifies, for every  $p \in \mathfrak{R}_{++}^n$ , exactly one element of  $\mathfrak{R}^n$ .

Given a DSF,  $S$ , and given any price vector  $p$ ,  $S(p)$  is the net output vector (uniquely) chosen. A DSF can evidently be identified with a *degenerate* SSF.<sup>4</sup>

An SSF may be used as an analytical construct to aggregate deterministic supply behavior by  $m$  competitive firms, represented by  $m$  (possibly different) DSFs, all facing the same price vector,  $p$ . Group (industry) supply can be modeled via a ‘representative’ competitive firm facing  $p$  and choosing according to an SSF such that, for any subset  $A$  of  $\mathfrak{R}^n$ , the probability of choosing a net output vector from  $A$  is simply the proportion of firms who do so.

**Definition 2.3.** For all  $j \in M = \{1, \dots, m\}$ , let  $S_j$  be a given DSF ( $S_1, \dots, S_m$  need not all be distinct). We say that an SSF,  $s$ , *aggregates*  $\langle S_1, \dots, S_m \rangle$  iff, for every  $p \in \mathfrak{R}_{++}^n$ , and every  $A \subseteq \mathfrak{R}^n$ ,  $[t(A) = \frac{|\{i \in M \mid S_i(p) \in A\}|}{m}]$ , where  $t = s(p)$ .

An SSF may also represent supply behavior of a single competitive firm subject to technology shocks. The intuitive interpretation here is that there exists a given  $m$ -tuple of technology sets (collections of feasible net output vectors), say  $\langle G_1, \dots, G_m \rangle$ . If  $G_j$  happened to be the technology set actually facing the firm, the firm would choose according to some DSF  $S_j$ . Given a price vector  $p$ , some exogenous process (‘nature’) randomly determines which technology set is

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<sup>4</sup> An SSF,  $s$ , is degenerate iff, for every  $p \in \mathfrak{R}_{++}^n$ , there exists  $x \in \mathfrak{R}^n$  such that  $t(\{x\}) = 1$ , where  $t = s(p)$ .

*realized*: the firm subsequently chooses according to the corresponding DSF. The probability that any technology set  $G_j$  will be realized is  $1/m$ , but since  $\langle G_1, \dots, G_m \rangle$  need not all be distinct, two distinct DSFs may have different probabilities of realization.

We now summarize, for later reference, the classical theory of competitive firm behavior.

**Definition 2.4.** A DSF,  $S$ , satisfies the *Weak Axiom of Profit Maximization* (WAPM) iff, for every ordered pair of price vectors  $\langle p, p' \rangle$ ,  $[p(x - x') \geq 0]$ , where  $x = S(p), x' = S(p')$ .

WAPM requires that, if a competitive firm happens to choose some net output vector  $x$  when faced with the price vector  $p$ , then it cannot choose any net output vector under another price situation which would give it a higher profit under  $p$ . The firm's supply function,  $S$ , can be rationalized, i.e. interpreted, as being driven by the goal of profit-maximization, if one can construct a set of net output vectors, say  $\Gamma$ , such that, if the firm's technology set was indeed  $\Gamma$ , and it wished to maximize its profit, then it would be able to do so by choosing according to  $S$ .<sup>5</sup> A *closed and convex* set  $\Gamma \subseteq \mathfrak{R}^n$  exists which rationalizes  $S$  in terms of profit maximization if, and only if,  $S$  satisfies WAPM.<sup>6</sup>

### 3. Stochastic consistency and the law of supply

We now introduce our consistency restriction for an SSF. Let  $x^* \in \mathfrak{R}^n$  be an arbitrary net output vector, and suppose the price vector changes from some initial situation  $p$  to  $p'$ . Consider the collection of all net output vectors whose *attractiveness relative to the reference vector  $x^*$  does not decline* due to the price change. This is the set of all net output vectors which continue to yield at least as much profit (or as low a loss) *relative to  $x^*$* , despite the price change. Thus, this collection consists of all  $x \in \mathfrak{R}^n$  which satisfy  $[p'(x - x^*) \geq p(x - x^*)]$ . It seems reasonable to require that a net output vector be chosen from this collection at least as frequently as earlier: the price shift has not reduced the attractiveness of its members. Thus, the probability that a net output vector is chosen from this set

<sup>5</sup> A DSF,  $S$ , is *rationalizable in terms of profit maximization with respect to a technology set* iff there exists some  $\Gamma \subseteq \mathfrak{R}^n$  such that, for all  $p \in \mathfrak{R}_{++}^n$ , (i)  $S(p) \in \Gamma$ , and (ii)  $[[pS(p) \geq pv'] \text{ for all } v' \in \Gamma]$ . The set  $\Gamma$  is said to *rationalize  $S$*  in terms of profit maximization.

<sup>6</sup> See Varian (1984).

should not decline. An analogous condition should hold for the set of net output vectors which are made *strictly more attractive* by the price shift.<sup>7</sup>

**Definition 3.1.** An SSF,  $s$ , satisfies *stochastic consistency* iff, for every pair of price vectors  $\langle p, p' \rangle$ , and for every  $x^* \in \mathfrak{R}^n$ :

$$t'(\{x \in \mathfrak{R}^n \mid p'(x - x^*) \geq p(x - x^*)\}) \geq t(\{x \in \mathfrak{R}^n \mid p'(x - x^*) \geq p(x - x^*)\}), \quad (1)$$

and

$$t'(\{x \in \mathfrak{R}^n \mid p'(x - x^*) > p(x - x^*)\}) \geq t(\{x \in \mathfrak{R}^n \mid p'(x - x^*) > p(x - x^*)\}). \quad (2)$$

where  $t = s(p)$  and  $t' = s(p')$ .

Stochastic consistency is the probabilistic analogue of a restriction on an individual firm's deterministic supply behavior, introduced by Dasgupta (2005), which we term 'consistency' here. Suppose the firm chooses, respectively, the net output vectors  $\tilde{x}, \tilde{x}'$ , under  $p, p'$ . Suppose further that, by choosing  $\tilde{x}$  instead of the feasible alternative  $\tilde{x}'$  under  $p$ , the firm loses some amount, say \$10. Consistency requires that the loss entailed by the choice of  $\tilde{x}$  instead of  $\tilde{x}'$  under  $p'$  must be at least \$10 (since otherwise, intuitively, a consistent firm should have persisted with  $\tilde{x}$  under  $p'$ , instead of switching to  $\tilde{x}'$ ).

**Definition 3.2.** A DSF,  $S$ , satisfies *consistency* iff, for every pair of price vectors  $\langle p, p' \rangle$ :

$$p'(\tilde{x}' - \tilde{x}) \geq p(\tilde{x}' - \tilde{x}); \quad (3)$$

where  $\tilde{x}' = S(p')$ ,  $\tilde{x} = S(p)$ .

Since (3) is equivalent to  $(p' - p)(\tilde{x}' - \tilde{x}) \geq 0$ , consistency is equivalent to the familiar Supply Inequality, which simply imposes this latter restriction.

**Observation 3.3.** A degenerate SSF,  $s$ , satisfies stochastic consistency iff the DSF corresponding to  $s$  satisfies consistency.

**Proof:** See the Appendix.

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<sup>7</sup> Intuitively, our condition thus treats profit as a consideration in choice, but not necessarily the only consideration.

Dasgupta has shown that, for a DSF, consistency is weaker than WAPM (recall Definitions 3.2 and 2.4 and note Example 3.7 below). By Observation 3.3, the following relationship thus holds between WAPM and stochastic consistency for a degenerate SSF.

**Observation 3.4.**

- (i) A degenerate SSF,  $s$ , may satisfy stochastic consistency even if the DSF corresponding to  $s$  violates WAPM.
- (ii) If a DSF,  $S$ , satisfies WAPM, then the degenerate SSF corresponding to  $S$  must satisfy stochastic consistency.

When an SSF aggregates a finite class of DSFs (recall Definition 2.3), stochastic consistency turns out to be a weaker restriction than the condition that the constituent DSFs all individually satisfy consistency.

**Lemma 3.5.** Let  $M = \{1, \dots, m\}$  be a given finite set,  $|M| \geq 1$ .

- (i) For every  $m$ -tuple of DSFs  $\langle S_1, \dots, S_m \rangle$  such that [for all  $j \in M$ ,  $S_j$  satisfies consistency], the SSF aggregating  $\langle S_1, \dots, S_m \rangle$  satisfies stochastic consistency.
- (ii) For all  $m \geq 2$ , there exists an  $m$ -tuple of DSFs  $\langle S_1, \dots, S_m \rangle$  such that: [for every  $j \in M$ ,  $S_j$  violates consistency], but the SSF aggregating  $\langle S_1, \dots, S_m \rangle$  satisfies stochastic consistency.

**Proof:** See the Appendix.

Recall now that, for a DSF, WAPM necessarily implies consistency; but a DSF may satisfy consistency, yet violate WAPM (see Example 3.7 below). In light of this, Lemma 3.5 immediately yields the following.

**Proposition 3.6.** Let  $M = \{1, \dots, m\}$  be a given finite set,  $|M| \geq 1$ .

- (i) For every  $m$ -tuple of DSFs  $\langle S_1, \dots, S_m \rangle$  such that [for all  $j \in M$ ,  $S_j$  satisfies WAPM], the SSF aggregating  $\langle S_1, \dots, S_m \rangle$  satisfies stochastic consistency.
- (ii) There exists an  $m$ -tuple of DSFs,  $\langle S_1, \dots, S_m \rangle$ , such that: [for every  $j \in M$ ,  $S_j$  violates WAPM], but the SSF aggregating  $\langle S_1, \dots, S_m \rangle$  satisfies stochastic consistency.

Proposition 3.6 summarizes our central finding. Proposition 3.6(ii) brings into focus the disconnect between profit maximization and stochastic consistency:

the aggregate SSF representation may satisfy stochastic consistency even if *not a single* underlying DSF can be rationalized via profit-maximization. Thus, given any finite collection of DSFs, stochastic consistency for their aggregate SSF representation is weaker than the requirement that all constituent DSFs be rationalizable in terms of profit maximization.

Our framework takes, as its theoretical prior, a *given* finite collection of DSFs, and derives an SSF as its aggregate representation. This seems the natural modeling approach in the case of aggregation over a collection of competitive firms. In the case of an SSF that models supply behavior of an individual firm subject to stochastic technology shocks, in some empirical contexts, the underlying collection of alternative DSFs over which ‘nature’ randomizes may not be directly observable. Proposition 3.6(ii) implies that one can have a *given* prior collection of DSFs, all of whom violate WAPM, yet whose aggregate SSF representation satisfies stochastic consistency. But, given an SSF,  $s$ , satisfying stochastic consistency, does there necessarily exist *some*  $m$ -tuple of DSFs, at least one of whom satisfies WAPM, and whose aggregate representation is  $s$ ?

The following example shows that even this is not the case. An SSF,  $s$ , may satisfy stochastic consistency, yet *every* possible finite collection of DSFs that yields  $s$  as its aggregate representation may exhibit violation of WAPM by *all* its constituents. Thus, an SSF may satisfy stochastic consistency even when it is completely impervious to *ex post* rationalization in terms of profit-maximization.

**Example 3.7.** Let  $s$  be an SSF specified as follows: for all  $p \in \mathfrak{R}_{++}^n$ ,  

$$[t(\{(1,-1,0,\dots,0)\}) = t(\{(1,-2,0,\dots,0)\}) = \frac{1}{2} \quad \text{if} \quad p_2 \geq 1; \quad \text{and}$$

$$t(\{(1,-3,0,\dots,0)\}) = t(\{(1,-4,0,\dots,0)\}) = \frac{1}{2} \quad \text{if} \quad p_2 < 1], \quad \text{where} \quad t = s(p).$$
 The SSF  $s$  satisfies stochastic consistency. Nonetheless, any  $m$ -tuple of DSFs that  $s$  can be said to aggregate must exhibit violation of WAPM (and satisfaction of consistency) by all its constituents.

**Remark 3.8.** The example above also shows that an SSF may satisfy stochastic consistency, yet its constituent DSFs may all violate *cost-minimization*. Furthermore, an SSF may satisfy stochastic consistency even when it is inconsistent with any collection of DSFs satisfying cost-minimization.

An SSF,  $s$ , satisfies stochastic consistency whenever there exists at least one  $m$ -tuple of DSFs that  $s$  can be said to aggregate, all of whose members individually satisfy consistency (Lemma 3.5(i)). Intuitively, economizing on input use when input prices are high, but not when they are low, does not appear

intrinsically contradictory, even though this entails losses in the latter situation. X-efficiency analyses of firm behavior, for example, often highlight cases where firms improve their technical efficiency in response to a rise in input costs. Evidently, every member of any  $m$ -tuple of DSFs that the SSF in Example 3.7 can be said to aggregate must exhibit this feature. Consistency permits such internally coherent loss-making behavior, WAPM rules it out. In essence, it is this difference between the two conditions that Example 3.7 exploits.<sup>8</sup>

We now specify a probabilistic analogue of the deterministic Supply Inequality, which summarizes the main empirical content of the classical theory of competitive firm behavior.

**Definition 3.9.** An SSF,  $s$ , satisfies *stochastic supply inequality* iff, for every pair of price vectors  $\langle p, p' \rangle$ , and for every  $z \in \mathfrak{R}$  :

$$t'(\{x \in \mathfrak{R}^n \mid (p' - p)x \geq z\}) \geq t(\{x \in \mathfrak{R}^n \mid (p' - p)x \geq z\}), \quad (4)$$

and

$$t'(\{x \in \mathfrak{R}^n \mid (p' - p)x > z\}) \geq t(\{x \in \mathfrak{R}^n \mid (p' - p)x > z\}); \quad (5)$$

where  $t = s(p)$  and  $t' = s(p')$ .

Let  $z$  be any arbitrary real number, and suppose the price vector changes from some initial situation  $p$  to  $p'$ . Consider the collection of all net output vectors whose profitability increases by at least  $z$  due to the price change. Stochastic supply inequality requires that the probability of choosing from this collection should not fall under  $p'$ . An analogous requirement should also hold with regard to the collection of all net output vectors whose profitability increases by more than  $z$ .

It can be checked that stochastic supply inequality is equivalent to stochastic consistency. This general equivalence evidently subsumes the equivalence of consistency and supply inequality for the deterministic

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<sup>8</sup> Satisfaction of stochastic consistency by an SSF,  $s$ , is necessary to ensure the existence of some finite collection of DSFs that yields  $s$  as its aggregate representation, while exhibiting satisfaction of consistency by all its constituents (recall Lemma 3.5(i)). But is it also sufficient? This important question appears open at this stage. Given any pair of price vectors  $\langle p, p' \rangle$ , and an SSF,  $s$ , satisfying stochastic consistency, there must exist an  $m$ -tuple of DSFs, whose aggregate representation is  $s$ , and whose members all individually satisfy (3) for  $\langle p, p' \rangle$ . But does there necessarily exist at least one such  $m$ -tuple, whose members all individually satisfy (3) for *every other* possible pair of price vectors as well? We have been unable to resolve this issue so far.

(degenerate) special case. Furthermore, in light of this general equivalence, stochastic consistency yields a probabilistic expansion of the law of supply, which subsumes the traditional, deterministic, version.

Suppose the price of a commodity rises, all other prices remaining invariant. Our (stochastic dominance) formulation of the law of supply requires that, for every real number  $\beta$ , neither the probability of producing at least  $\beta$  amount of the commodity, nor the probability of producing more than  $\beta$  amount of the commodity, should decrease.

**Notation 3.10.** For every  $i \in N$ , let  $K_i$  denote the set of all ordered pairs of price vectors  $\langle p, p' \rangle$  such that (i)  $[p_j = p'_j \text{ for all } j \in N \setminus \{i\}]$  and (ii)  $p'_i > p_i$ .

**Definition 3.11.** An SSF,  $s$ , satisfies the *law of supply* iff, for all  $\beta \in R$ , for all  $i \in N$ , and for all ordered pairs  $\langle p, p' \rangle \in K_i$ , we have:

$$t'(\{x \in \mathfrak{R}^n \mid x_i \geq \beta\}) \geq t(\{x \in \mathfrak{R}^n \mid x_i \geq \beta\}), \quad (6)$$

and

$$t'(\{x \in \mathfrak{R}^n \mid x_i > \beta\}) \geq t(\{x \in \mathfrak{R}^n \mid x_i > \beta\}); \quad (7)$$

where  $t = s(p)$  and  $t' = s(p')$ .

**Proposition 3.12.** If an SSF satisfies stochastic consistency, then it must satisfy the law of supply.

**Proof:** See the Appendix.

The law of supply in its standard, deterministic, version follows from Proposition 3.12.

**Corollary 3.13.** If a DSF,  $S$ , satisfies consistency, then, for all  $i \in N$ , and for all ordered pairs  $\langle p, p' \rangle \in K_i$ ,  $[x'_i \geq x_i]$ , where  $x = S(p)$ ,  $x' = S(p')$ .

#### 4. Concluding remarks

This paper extends and completes a line of investigation initiated by Becker (1962) and continued by Dasgupta (2005), which argued that the primary empirical/predictive content of the traditional theory of competitive firm behavior can be delinked from its behavioral presumption of profit-maximization. Our analysis provides a necessary and sufficient axiomatic foundation for such de-

linkage, while subsuming both the contribution of Dasgupta and the traditional, WAPM-based theory as special cases.<sup>9</sup>

We have utilized the analytical construct of a stochastic supply function to provide an aggregate representation of a finite class of standard deterministic supply functions. We have introduced an intuitively plausible consistency postulate for a stochastic supply function that may be satisfied even if no underlying deterministic supply function is open to rationalization in terms of profit maximization (nor, indeed, satisfies cost-minimization). In this sense, our consistency postulate provides a complete conceptual departure from the traditional presumption of profit-maximization. Despite this departure, our consistency postulate turns out to be equivalent to a stochastic analogue of the deterministic condition of supply inequality, which summarizes the predictive content of the traditional theory of competitive firm behavior. This finding provides the central equivalence in the theory of competitive firm behavior, in that a number of results in the deterministic theory follow as special cases. In particular, it yields a probabilistic version of the law of supply, which implies the traditional specification. It is difficult to see how a theory of supply with any applicability can afford to dispense with the law of supply, at least in its probabilistic version. In this sense, our analysis also appears to set the conceptual limits beyond which the behavioral presumptions of the traditional theory cannot be substantively relaxed without seriously undermining its predictive import.

If not profit, exactly what is the objective function (if any) that a competitive firm need maximize to satisfy our consistency condition? In other words, can one characterize some objective function, maximization of which over probabilistic convex technology sets would provide a necessary and sufficient rationalization of firm choice behavior that satisfies our stochastic consistency? Would such an objective function be open to intuitive interpretation? These questions, which relate to a preference-based counterpart of the choice-based theory of competitive supply developed in this paper, suggest themselves as useful candidates for future investigations.

## Appendix

### Proof of Observation 3.3.

First consider a degenerate SSF,  $s$ , satisfying stochastic consistency. Since  $s$  is degenerate, there must exist some  $\tilde{x}', \tilde{x} \in \mathfrak{R}^n$  such that  $t(\{\tilde{x}'\}) = 1 = t(\{\tilde{x}\})$ . Since, given stochastic consistency, (1) holds for all  $x^* \in \mathfrak{R}^n$ , it must also hold for  $x^* \equiv \tilde{x}$ . Thus, (1) implies:  $[t(\{x \in \mathfrak{R}^n \mid p'(x - \tilde{x}) \geq p(x - \tilde{x})\}) = 1]$ ; i.e., (3).

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<sup>9</sup> In so doing, we also provide a supply theoretic parallel to the revealed preference treatment of stochastic demand behavior recently developed by Bandyopadhyay et al. (2004, 2002, 1999).

Now consider a DSF,  $S$ , satisfying consistency. Given any  $x^* \in \mathfrak{R}^n$ , (3) is equivalent to:

$$(p' - p)(\tilde{x}' - x^*) \geq (p' - p)(\tilde{x} - x^*). \quad (8)$$

The degenerate SSF corresponding to  $S$  must therefore satisfy, for all  $x^* \in \mathfrak{R}^n$ ,

$$t(\{x \in \mathfrak{R}^n \mid (p' - p)(x - x^*) \geq 0\}) \geq t(\{x \in \mathfrak{R}^n \mid (p' - p)(x - x^*) \geq 0\}); \quad (9)$$

$$t(\{x \in \mathfrak{R}^n \mid (p' - p)(x - x^*) > 0\}) \geq t(\{x \in \mathfrak{R}^n \mid (p' - p)(x - x^*) > 0\}). \quad (10)$$

Respectively, (9) and (10) imply (1) and (2).  $\diamond$

**Proof of Lemma 3.5.**

(i) Suppose, for all  $j \in M$ ,  $S_j$  satisfies consistency. Consider any  $p, p' \in \mathfrak{R}^n$  and any  $x^* \in \mathfrak{R}^n$ . Let  $S_j(p) = \tilde{x}^j$ ,  $S_j(p') = \tilde{x}^{j'}$ . Then, from (3), we get:

$$\text{for all } j \in M, [(p' - p)(\tilde{x}^{j'} - x^*) \geq (p' - p)(\tilde{x}^j - x^*)]. \quad (11)$$

Notice now that the aggregate SSF representation,  $s$ , of  $\langle S_1, \dots, S_m \rangle$  must satisfy:

$$t(\{x \in \mathfrak{R}^n \mid (p' - p)(x - x^*) \geq 0\}) = \frac{|\{j \in M \mid (p' - p)(S_j(p) - x^*) \geq 0\}|}{m};$$

$$t'(\{x \in \mathfrak{R}^n \mid (p' - p)(x - x^*) \geq 0\}) = \frac{|\{j \in M \mid (p' - p)(S_j(p') - x^*) \geq 0\}|}{m}.$$

By (11),

$$\{j \in M \mid (p' - p)(S_j(p) - x^*) \geq 0\} \subseteq \{j \in M \mid (p' - p)(S_j(p') - x^*) \geq 0\}.$$

Condition (1) follows. An analogous argument establishes (2).

(ii) Let  $p_i$  be the price of commodity  $i \in N$ , and let the net output vector supplied according to the DSF  $S_j$  be denoted by  $x^j$ ,  $j \in M$ . Let  $x^1 = (1, -1, 0, \dots, 0)$  if either  $1 > p_2$  or  $p_2 \geq m$ ;  $x^1 = (1, -3, 0, \dots, 0)$  otherwise. For all

$j \in \{2, \dots, m\}$ , let  $x^j = (1, -1, 0, \dots, 0)$  if  $j > p_2 \geq j-1$ ;  $x^j = (1, -3, 0, \dots, 0)$  otherwise. Then, for every  $j \in M$ ,  $S_j$  violates consistency. Consider the SSF aggregation,  $s$ , of  $\langle S_1, \dots, S_m \rangle$ : regardless of the price vector, the net output vectors  $(1, -1, 0, \dots, 0)$  and  $(1, -3, 0, \dots, 0)$  must be chosen with probability  $\frac{1}{m}$  and  $\frac{m-1}{m}$ , respectively. Clearly,  $s$  satisfies stochastic consistency.  $\diamond$

**Proof of Proposition 3.12.**

Noting (4), if an SSF satisfies stochastic supply inequality, for all  $i \in N$ , for all  $\langle p, p' \rangle \in K_i$ , and for every  $z \in \mathfrak{R}$ :

$$t' \left\{ \left\{ x \in \mathfrak{R}^n \mid x_i \geq \frac{z}{(p'_i - p_i)} \right\} \right\} \geq t \left\{ \left\{ x \in \mathfrak{R}^n \mid x_i \geq \frac{z}{(p'_i - p_i)} \right\} \right\}. \quad (12)$$

Hence, given any  $\beta \in \mathfrak{R}$ , (12) holds for  $z = \beta(p'_i - p_i)$ : (6) follows. Noting (5), an analogous argument establishes (7). Proposition 3.12 follows from the equivalence between stochastic supply inequality and stochastic consistency.  $\diamond$

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