Pages 1-4 are from Wikipedia – selected & edited by Prof K, April 29, 2008.

Scientific method refers to the body of techniques for investigating phenomena, acquiring new knowledge, or correcting and integrating previous knowledge. It is based on gathering observable, empirical and measurable evidence subject to specific principles of reasoning. A scientific method consists of the collection of data through observation and experimentation, and the formulation and testing of hypotheses.

Theory

In science, a theory is a **mathematical** or logical explanation, or a testable model of the manner of interaction of a set of natural phenomena, capable of predicting future occurrences or observations of the same kind, and capable of being tested through experiment or otherwise verified through empirical observation.

In common usage, the word **theory** is often used to signify a conjecture, an opinion, or a speculation. In this usage, a theory is not necessarily based on facts; in other words, it is not required to be consistent with true descriptions of reality. This usage of theory leads to the common incorrect statements. True descriptions of reality are more reflectively understood as statements which would be true independently of what people think about them.

According to the National Academy of Sciences,
"Some scientific explanations are so well established that no new evidence is likely to alter them. The explanation becomes a scientific theory. ***In everyday language a theory means a hunch or speculation. Not so in science.*** *In science, the word* ***theory refers to a comprehensive explanation of an important feature of nature that is supported by many facts*** *gathered over time. Theories also allow scientists to make predictions about as yet unobserved phenomena."*

In scientific usage, a *theory* does not mean an unsubstantiated *guess* or *hunch*, as it can in everyday speech. A theory is a logically self-consistent **model** or framework for describing the behavior of a related set of natural or social phenomena. In this sense, **a theory is a systematic and formalized expression of all previous observations, and is predictive, logical, and testable.**

The term *theoretical*

The term *theoretical* is sometimes informally used in lieu of *hypothetical* to describe a result which is predicted by theory but has not yet been adequately tested by observation or experiment. It is not uncommon for a theory to produce predictions which are later confirmed or proven incorrect by experiment. By inference, a prediction proved incorrect by experiment demonstrates that the hypothesis is invalid. This either means the theory is incorrect or that the experimental conjecture was wrong and the theory did not predict the hypothesis.

A **hypothesis** (from Greek *ὑπόθεσις*) consists either of a suggested explanation for a phenomenon or of a reasoned proposal suggesting a possible correlation between multiple phenomena.

The scientific method requires that one can test a **scientific hypothesis**.

Even though the words "hypothesis" and "theory" are often used synonymously in common and informal usage, a scientific *hypothesis* is not the same as a scientific *theory*.

Normally, scientific hypotheses have the form of a mathematical model.

Mathematical models

A **mathematical model** is an abstract model that uses mathematical language to describe a system. Mathematical models are used particularly in the natural sciences and engineering disciplines (such as physics, biology, and electrical engineering) but also in the social sciences (such as economics, sociology and political science). Physicists, engineers, computer scientists, and economists use mathematical models most extensively.

Classifying mathematical models

Many mathematical models can be classified in some of the following ways:

Linear vs. nonlinear: Mathematical models are usually composed by **variables**, which are abstractions of quantities of interest in the described systems, and operators that act on these variables, which can be algebraic operators, **functions**, differential operators, etc. If all the operators in a mathematical model present linearity, the resulting mathematical model is defined as linear. A model is considered to be nonlinear otherwise.

The question of linearity and nonlinearity is dependent on context, and linear models may have nonlinear expressions in them. For example, in a **statistical linear model**, it is assumed that a relationship is linear in the parameters, but it may be nonlinear in the predictor variables.

In a mathematical programming model (for example, the constrained optimization approach of **Lagrange**), if the objective functions and constraints are represented entirely by linear equations, then the model is regarded as a linear model. If one or more of the objective functions or constraints are represented with a nonlinear equation, then the model is known as a nonlinear model.

A common approach to nonlinear problems is **linearization** (it's inappropriate to linearize if you are studying phenomena such as *irreversibility* which are strongly associated with nonlinearity).

Two examples of mathematical models

1) Model of rational behavior of a consumer

To model rational consumer behavior we assume that a consumer faces a choice of n commodities labeled 1,2,...,n each with a market price p_1, p_2,..., p_n. The consumer is assumed to have a utility function U depending on the amounts of commodities x_1, x_2,..., x_n consumed. Further, the consumer has a budget M which she uses to purchase a vector x_1, x_2,..., x_n in such a way as to maximize $U(x_1, x_2,..., x_n)$.

The problem of rational behavior in this model then becomes an optimization problem, that is

$$\max U(x_1, x_2, \ldots, x_n)$$

subject to:

$$\sum_{i=1}^{n} p_i x_i \leq M.$$

$$x_i \geq 0 \quad \forall i \in \{1, 2, \ldots, n\}$$

2) ***Population Growth*****.** The preferred population growth model is the **logistic function**.

The Verhulst equation

[edit]

A typical application of the logistic equation is a common model of population growth, which states that:

- the rate of reproduction is proportional to the existing population, all else being equal
- the rate of reproduction is proportional to the amount of available resources, all else being equal. Thus the second term models the competition for available resources, which tends to limit the population growth.

Letting P represent population size (N is often used in ecology instead) and t represent time, this model is formalized by the differential equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

where the constant r defines the growth rate and K is the carrying capacity. In ecology, species are sometimes referred to as r-strategist or K-strategist depending upon the selective processes that have shaped their life history strategies. The solution to the equation (with P_0 being the initial population) is

$$P(t) = \frac{KP_0e^{rt}}{K + P_0\left(e^{rt} - 1\right)}$$

where

$$\lim_{t\to\infty} P(t) = K.$$

Sigmoid function

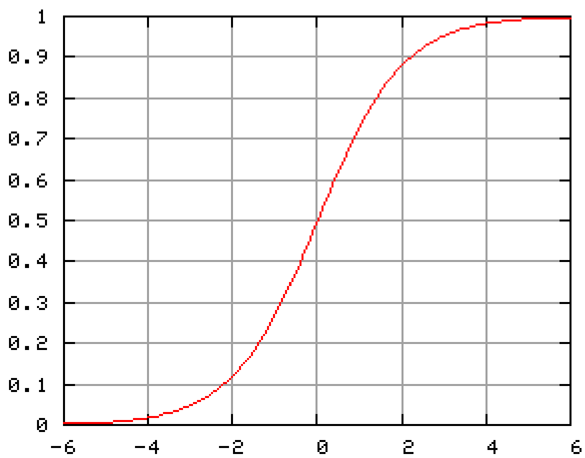
[edit]

Main article: sigmoid function

The special case of the logistic function with $a = 1$, $m = 0$, $n = 1$, $\tau = 1$, namely

$$P(t) = \frac{1}{1 + e^{-t}}$$

is called **sigmoid function** or **sigmoid curve**. The name is due to the sigmoid shape of its graph. This function is also called the standard logistic function and is often encountered in many technical domains, especially in artificial neural networks as a transfer function, probability, statistics, biomathematics, mathematical psychology and economics.



Logistic curve, specifically the sigmoid function

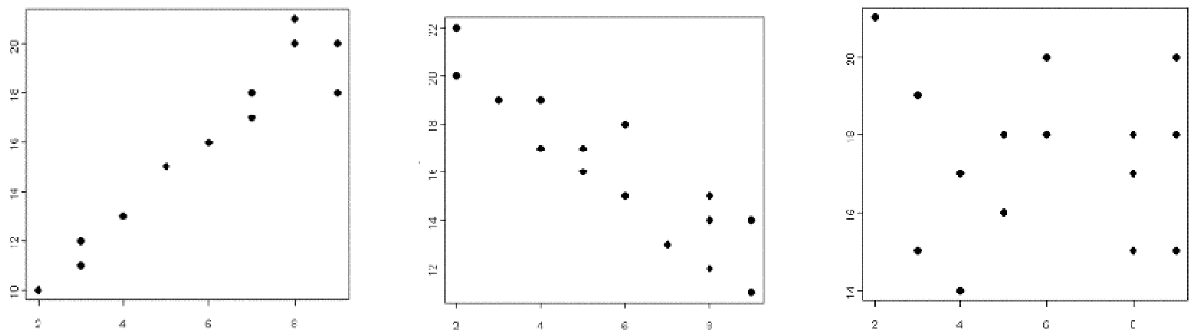
Model Evaluation

A crucial part of the modeling process is the evaluation of whether or not a given mathematical model describes a system accurately.

Fit to Empirical Data

Usually the easiest part of model evaluation is checking whether a model fits experimental measurements or other empirical data. In models with parameters, a common approach to test this fit is to split the data into two disjoint subsets: training data and verification data. The training data are used to estimate the model parameters. An accurate model will closely match the verification data even though this data was not used to set the model's parameters. This practice is referred to as *cross-validation* in statistics.

While it is rather straightforward to test the appropriateness of parameters, it can be more difficult to test the validity of the general mathematical form of a model. In general, more mathematical tools have been developed to test the fit of statistical models than models involving Differential equations.



For economic models, the **null hypothesis** (H_0) is what we expect if there is no relationship between the dependent and hypothetically or theoretically explanatory variable(s) in our model. The null hypothesis must be refuted in order to support an *alternative hypothesis*.

We presume that the null hypothesis is true until **statistical evidence**, in the form of a hypothesis test, indicates otherwise — that is, when the researcher has a certain degree of confidence, usually 95% to 99%, that the data does not support the null hypothesis.

The “prob” or “p-value” of an estimated coefficient is the probability that the null hypothesis is true; or that the coefficient is zero, which indicates that there is no relationship between the presumed explanatory variable and the outcome or dependent variable.

Type I error, an **α error**, or a "**false positive**" is the mistake of rejecting a ***null hypothesis*** when it is actually true. (We incorrectly think we found a relationship when there really isn’t one.)

An analogy to a ‘Type I error” is when a woman’s pregnancy test says she IS pregnant (reject the null) when in fact she’s not pregnant.

Type II error, a **β error**, or a "**false negative**" is the mistake of failing to reject a null hypothesis. (Our alternative hypothesis might be true, but we incorrectly accept the null instead.)

The analogy to a Type II error is when the pregnancy test indicates that a woman is not pregnant (accept the null) when in fact she is.

Prof K's "5 Question Approach" to formalizing an *economic model*:

0. what is the issue/problem?

1. **WHO** is involved?

   (people as consumers, business people, employers, employees, public officials, borrowers, lenders, recreators, …)

2. what are the **OBJECTIVES** of each involved *who*?

   (maximize <u>utility</u>, maximize <u>profit</u>, minimize <u>cost</u> to fulfill an order for Q, minimize deadweight losses,…)

3. what **INSTRUMENTS** can be chosen by each *who* to achieve their objective(s)? That is, what is *endogenous*?

   (the mix and level(s) of goods to buy (Q_D); the mix and level(s) of goods to produce and sell (Q_S), the number of people or inputs to employ (L,K), a tax or subsidy rate, the amount to loan or borrow, the location to open your business, …)

4. what **CONSTRAINS** each who from achieving unlimited amounts of their objective(s) – what can't they choose or can't change? That is, what is *exogenous*?

   (wages, prices, the market structure, technology, income or budget, preferences, distances, endowments,…)

5. what is the relevant **TIME HORIZON**?

   (The "long run" is the time within which <u>everything</u> can be changed – or--
   in the "long run," everything is a choice variable and nothing is a constraint.)

EXAMPLE:

$$\max U(x_1, x_2, \ldots, x_n)$$

subject to:

$$\sum_{i=1}^{n} p_i x_i \leq M.$$

$$x_i \geq 0 \quad \forall i \in \{1, 2, \ldots, n\}$$

Structural model *outcome = f(choice variables, exogenous variables; α,β,…)*

Pros: analysis provides insights

Correct assumptions about structure provide more accurate predictions

Cons: potential simultaneity bias

GiGo

Reduced form model *outcome = f(other variables)*

Pros: lets the data "speak"

Timing evidence *(post hoc, ergo propter hoc)*

Cons: correlation does not imply causation

Possibility of reverse causation

One man's lead is another's lag

Missing variable bias