KEY RECO 250 Homework #5 by Prof. Kilkenny
October 19, 2009

1. Find the *first*, *second,* and *third derivatives* of the following *univariate* functions:

   a) $y = f(x) = x^3$

   $f'(x) = 3x^2$

   $f''(x) = 6x$

   $f'''(x) = 6$

   b) $y = f(x) = (8x-4)^3$

   $f'(x) = 24(8x-4)^2$

   $f''(x) = 384(8x-4) = 3072x - 1536$

   $f'''(x) = 3072$

2. Use the *Implicit Function Theorem* to find $\frac{dy}{dx}$:

   a) $2x^4+7x^3+8y^5-136 = 0$

By the ImpT Th: $\frac{dy}{dx} = \frac{-F_x}{F_y} = \boxed{\frac{-(8x^3+21x^2)}{40y^4}}$

   b) $7x^4+3x^3y+9xy^2-496 = 0$

By the ImpT Th: $\frac{dy}{dx} = \frac{-F_x}{F_y} = \boxed{\frac{-(28x^3+9x^2y+9y^2)}{3x^3+18xy}}$

3. Use the *Inverse function rule* to find $\frac{dQ}{dP}$ given $P = 220 - 4Q$.

The *inverse function rule* says that given $P= f(Q)$, $\frac{dQ}{dP} = \frac{1}{\left(\frac{dP}{dQ}\right)}$

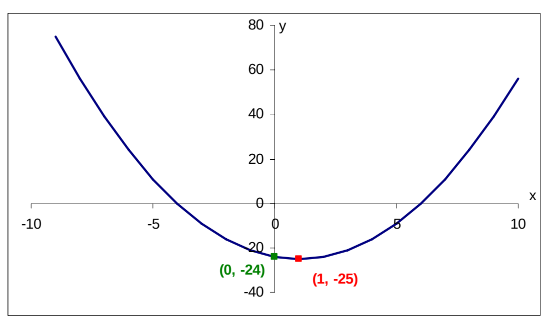
$$\frac{dP}{dQ} = -4 \quad \text{so} \quad \boxed{\frac{dQ}{dP} = -\frac{1}{4}}$$

4. Find the y-intercept, *relative extrema* ("x^*"), and *concavity* or *convexity*:

   a) $y = f(x) = x^2 - 2x - 24$

   $f(0) = 0^2 - 2\cdot 0 - 24 = -24$

→ *the y-intercept is (0, -24)*



f'(x) $= 2x - 2$

FONC: $f'(x^*) = 2x^* - 2 = 0$

$x^* = 1$

$f(x^*) = 1^2 - 2 - 24 = -1 - 24 = -25$

→ *the relative extremum* (or **critical point**) *is (1, -25)*

(and because there's only one wiggle, That's It.)

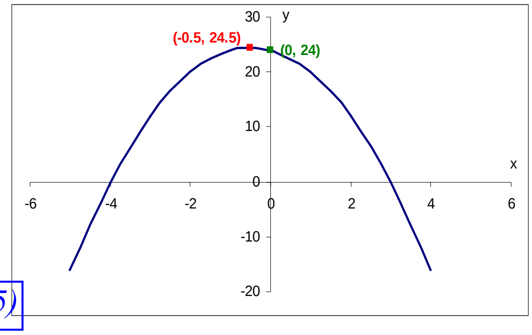
SOC: **f''(x)** $= 2 > 0$ *always positive; always "up"* → *the function is* **Convex**

Find the y-intercept, *relative extrema* ("x^*"), and *concavity* or *convexity*:

b) $y = f(x) = -2x^2 - 2x + 24$

$f(0) = -0^2 - 2\cdot 0 + 24 = +24$

→ *the y-intercept is (0, 24)*



$f'(x)$ $= -4x - 2$

FONC: $f'(x^*) = -4x^* - 2 = 0$

$x^* = -½$

$f(x^*) = -(½)^2 - 2(-½) + 24 = 24.5$

→ *the relative extremum* (or ***critical point***) *is (-½ , 24.5)*

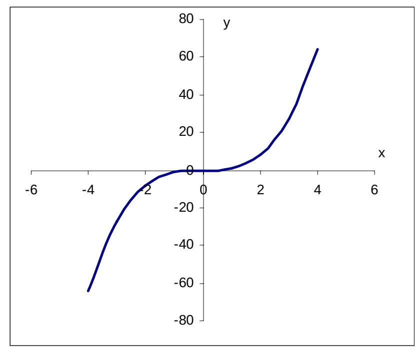
(and because there's only one wiggle, That's It.)

SOC: **$f''(x)$** $= -4$ *always negative; always "down"* → *the function is* ***Concave***

c) $y = f(x) = x^3$

$f(0) = 0^3 = 0$

→ *the y-intercept is (0, 0)*



$f'(x)$ $= 3x^2$

FONC: $f'(x^*) = 3x^{*2} = 0$

$x^* = 0$

$f(x^*) = 0$

→ *the relative extremum* (or ***critical point***) *is (0 , 0)*

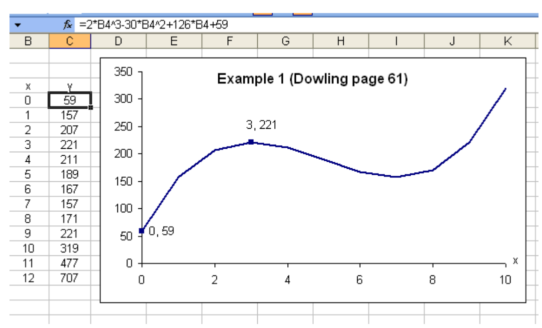
SOC: **$f''(x^*)$** $= 6x = 6(0) = 0$

Hmmm: this looks like an *inflection point* *(We KNOW it's NOT a linear function).*

Check the left side of x^*: **$f''(x^*-1)$** $= 6(0-1) = -6$ *<0, going down* ***→ concave on the left***

Check the right side of x^*: **$f''(x^*+1)$** $= 6(1) = 6$ *>0, going up* ***→ convex on the right***

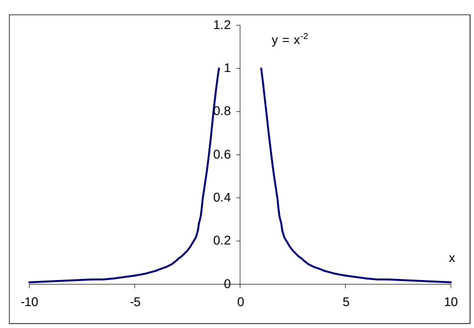
5. See EXAMPLE 1 on page 61 in the Dowling text. Illustrate (graph) the function.



6. create fully labeled separate illustrations (graphs) of the following functions (and turn them in).

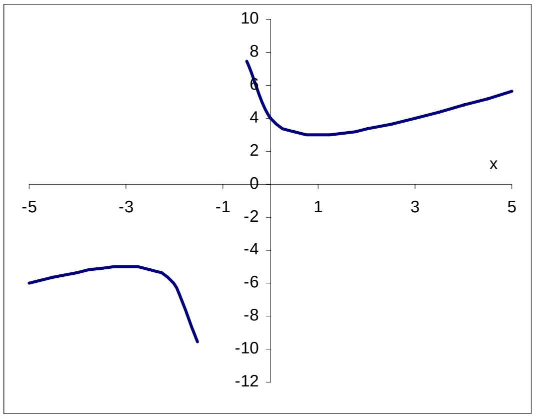
a) $y = \dfrac{1}{x^2}$ for the domain $x \subset [-10,10]$

HINT: clear any #DIV/0! cells before graphing

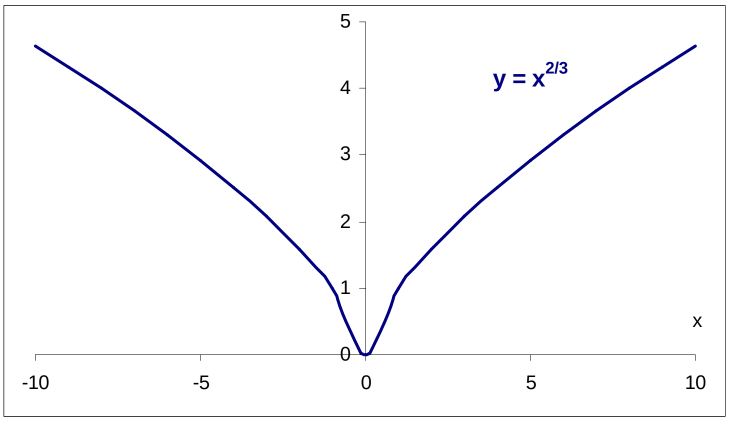


b) $y = x + \frac{4}{x+1}$ for the domain $x \subset [-5,5]$ with minor units of 0.5

HINT: clear any #DIV/0! cells before graphing



c) $y = x^{2/3}$ for the domain $x \subset [-10,10]$ **HINT**: write it as $(x^2)^{1/3}$



7. Using *first-order conditions*, find the optimal Q*. Verify w/ SOCs.

a) that maximizes profit given revenue, $R(Q)= 11Q - 2Q^2$, and cost, $C(Q) = Q^2+24$.

$\pi \equiv$ **Profit** $\equiv R(Q)-C(Q) = 11Q - 2Q^2 - [Q^2+24] = 11Q - 2Q^2 - Q^2 - 24 = \mathbf{11Q - 3Q^2 - 24}$

$\pi'(Q) = 11-6Q$

FONC: $\pi'(Q^*) = 11-6Q^* = 0$

$\boxed{Q^* = 11/6}$

SOC: $\pi''(Q) = -6 < 0$ *"down" at all Q*

→ *π(Q) is **concave***

*and **Q* is a global maximum**, as desired*

b) that minimizes average cost ("AC") given $C(Q) = Q^3 - 5Q^2 + 60Q$

AC $\equiv$ C(Q)/Q **= Q^2 -5Q + 60**

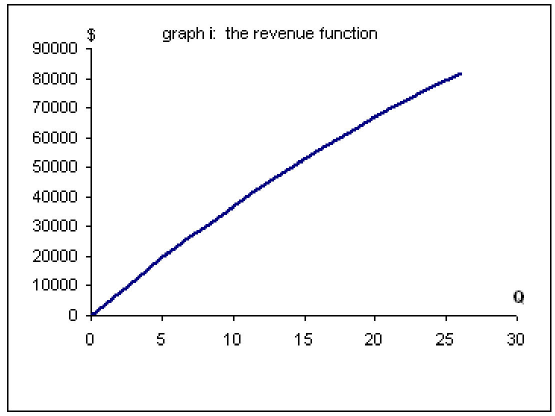
$$\frac{dAC}{dQ} = 2Q - 5$$

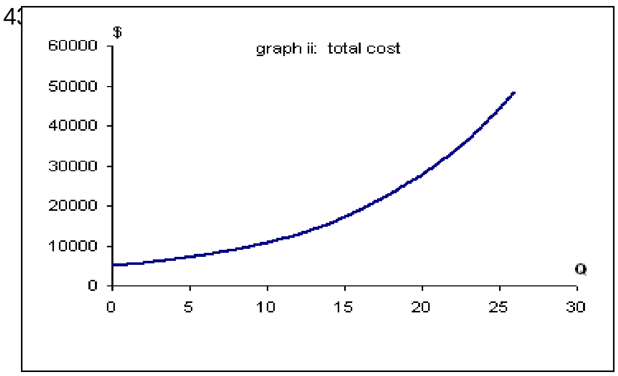
FONC: 2Q-5=0* → $\boxed{Q^* = 5/2}$

SOC: $\frac{d^2AC}{dQ^2} = 2 > 0$ *"up"* → *AC(Q) is **convex** at all Q, and **Q* is a global minimum**, as desired.*

8. Now use the information in EXAMPLE 4 on page 62 in the Dowling text. Use EXCEL, a graphing calculator, <u>or (best yet) by hand</u>, create FOUR separate illustrations/graphs of :

Q	revenue	cost	profit	AC	MC
0	0	5000	-5000		400
2	7868	5804	2064	2902	412
4	15472	6680	8792	1670	472
6	22812	7724	15088	1287.333	580
8	29888	9032	20856	1129	736
10	36700	10700	26000	1070	940
12	43248	12824	30424	1068.667	1192
14	49532	15500	34032	1107.143	1492
16	55552	18824	36728	1176.5	1840
18	61308	22892	38416	1271.778	2236
20	66800	27800	39000	1390	2680
22	72028	33644	38384	1529.273	3172
24	76992	40520	36472	1688.333	3712
26	81692	48524	33168	1866.308	4[illegible]





Marginal revenue
4000
3868