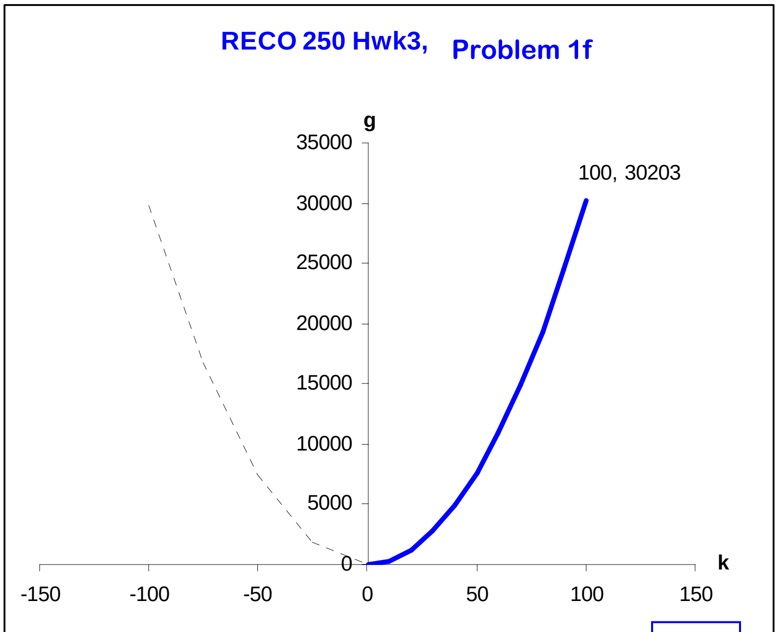
KEY RECO 250 Homework 3 Sept 15, 2009

1. Consider $g = f(k) = 3 + 2k + 3k^2$, defined for $k \subset [0,100]$
   a) What the *independent variable*? **k**
   b) what is the *domain*? **$k \subset [0,100]$**
   c) what is the *dependent variable*? **g**
   d) what is the *range* (i.e., what are the *minimum and maximum values* of the *dependent variable*)?
   minimum $g = f(\mathbf{k=0}) = 3 + 2\cdot0 + 3\cdot0^2 = 3$; maximum $g = f(\mathbf{k=100}) = 3 + 2\cdot100 + 3\cdot100^2 = 30{,}203$.
   So: <u>the range is **$g \subset [3,30203]$**</u>
   e) Evaluate the function at k=10.
   $f(k=10) = 3 + 2\cdot10 + 3\cdot10^2 =$ **323**.
   f) A graph of this function:



2. Evaluate $\sum_{k=1}^{4} 2^k = 2^1 + 2^2 + 2^3 + 2^4 = 2+4+8+16 = 30$

3. "add the first five integers" is written: $\sum_{x=1}^{5} x$

4. A business's costs of production are summarized by the following:
$$C(Q) = 120 + 45Q - Q^2 + 0.4Q^3$$
And the price customers will pay for each level it sells is summarized by the following market (inverse) demand curve:
$$P(Q) = 240 - 20Q.$$
Write out the business's PROFIT equation. Remember, Profit $\equiv P(Q)\cdot Q - C(Q)$.
(Extra credit: plot this on a graph with Q on the horizontal axis and \$profit on the vertical axis.)

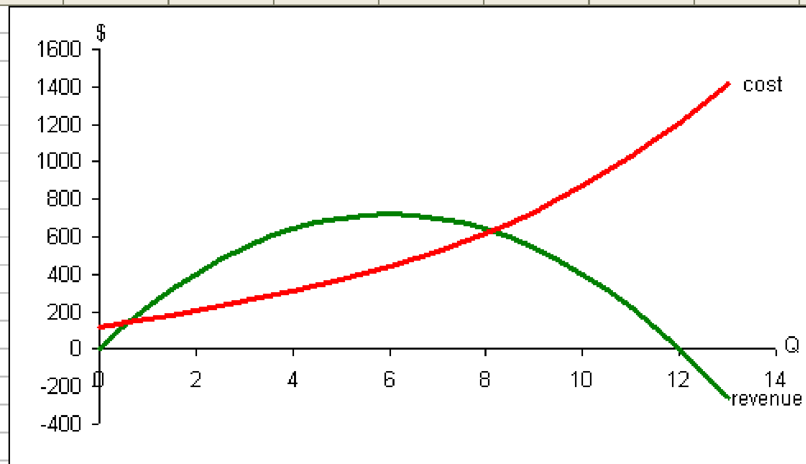
Profit $= P(Q)\cdot Q - C(Q) = (240 - 20Q)Q - [120 + 45Q - Q^2 + 0.4Q^3] = 240Q - 20Q^2 - 120 - 45Q + Q^2 - 0.4Q^3$

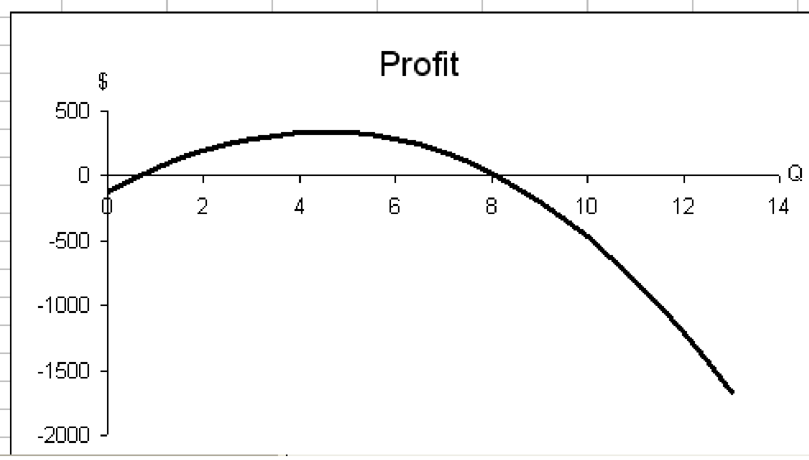
$$\text{Profit} = -0.4Q^3 - 19Q^2 + 195Q - 120.$$

The extra credit graph (definitely worth doing!) follows:

f_x =240*B19-20*B19^2

Q	$-0.4Q^3 - 19Q^2 + 195Q - 120$ profit	revenue	cost	verify: R-C
0	-120	0	120	-120
1	55.6	220	164.4	55.6
2	190.8	400	209.2	190.8
3	283.2	540	256.8	283.2
4	330.4	640	309.6	330.4
5	330	700	370	330
6	279.6	720	440.4	279.6
7	176.8	700	523.2	176.8
8	19.2	640	620.8	19.2
9	-195.6	540	735.6	-195.6
10	-470	400	870	-470
11	-806.4	220	1026.4	-806.4
12	-1207.2	0	1207.2	-1207.2
13	-1674.8	-260	1414.8	-1674.8



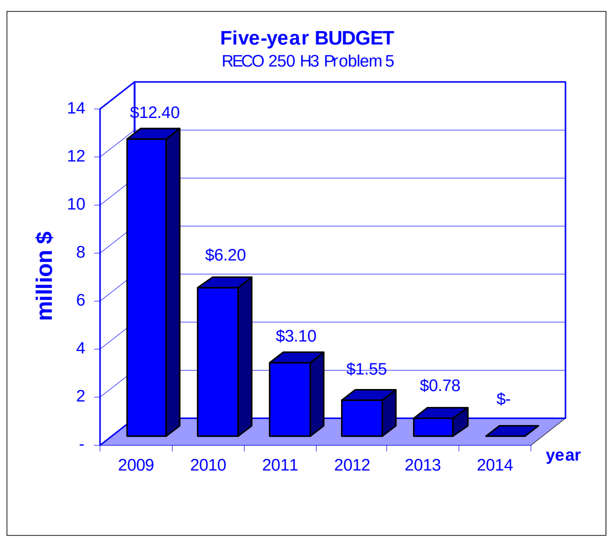


5. A government program with an annual budget of $12.4 million is to be eliminated over the next five years by cutting the budget by 50% each year (2010 to 2013) then to zero in 2014.

a) a table that lists the year, the level of the budget each year, and the amount reduced compared to the previous year:

BUDGET		
year	mil$	change ($mil)
2009	12.4	---
2010	6.2	-6.2
2011	3.1	-3.1
2012	1.55	-1.55
2013	0.775	-0.775
2014	0	-0.775

b) A graph of the trend in the budget:

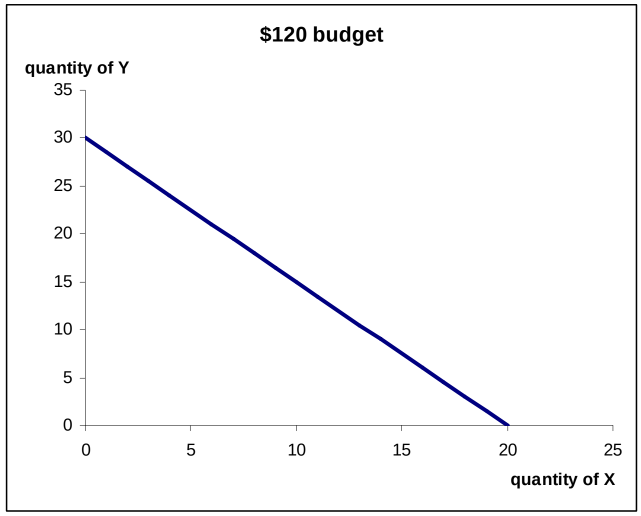


6. When $Q_S = -5 + 3P$ and $Q_d = 10 - 2P$, find the *equilibrium price* and *quantity*:

See page 15 for method: P* = 3 and Q* = 4

7. A person has \$120 to spend on items “X” and “Y.” Item X costs P_x=\$6 each, and P_y=\$4 each. If he spends all his money, he can afford various combinations or quantities of each, Q_x and Q_y. Write out the consumer’s budget equation such that his expenditure just equals \$120. Graph the expenditure identity (or *budget line*) with the Y good on the vertical axis, good X on the horizontal axis.

See page19-20 in the Dowling text for the method:
a budget is expressed generally as: $P_xX + P_yY = B$.



Given the prices and total expenditure (above),
the budget equation is: 6X + 4Y = 120

To graph it, rearrange this into a Y = b + mX “intercept, slope”
format: 4Y = 120 – 6X
Y = 30-1.5X

8. If demand is $Q_d = 2{,}160 - 180P_d$ and supply is $Q_S = -2{,}400 + 300P_S$, find the *equilibrium price* and *quantity*.

Step 1: equate demand and supply; and solve for the equilibrating price:

$$2{,}160 - 180P_d = -2{,}400 + 300P_S$$
$$2{,}160 + 2{,}400 = 180P + 300P$$
$$4{,}560 = 480P$$

\$9.50 = *P**

Step 2: find (either) Q at P*:

$$Q = 2{,}160 - 180 \cdot 9.5$$

***Q** *** **= 450**

9. Suppose consumers will buy 40 units of a product (per period) when the price is \$12.75 each; and 25 units when the price is \$18.75. Assume that the *demand function* is *linear*.

a) Find the (equation for) *demand function:*

Y = mX + b is a general linear equation
$Q_d = mP_d + b$ is the general form of a *demand function*

$$m \equiv slope \equiv \frac{\Delta Y}{\Delta X} = \frac{\Delta Q}{\Delta P} = \frac{40-25}{12.75-18.75} = \frac{15}{-6} = -2.5$$

$$b \equiv intercept: \quad Q_d = \frac{15}{-6}P_d + b$$

40 = -(15/6)·12.75 + b
71.88 = b

So, the demand equation is $\mathbf{Q_d = -2.5P_d + 71.88}$

b) what is the equation for the graph of demand?
To find this, invert the *demand function:*
$\mathbf{P_d = -0.4Q_d + 28.75}$

c) What price per unit are customers willing to pay for 37 units?
$\mathbf{P_d = -0.4 \cdot 37 + 28.75 = \$13.95}$

10. If *inverse* demand is $P_D = \frac{-7}{100}Q + 65$ and *inverse* supply is $P_S = \frac{8}{100}Q + 50$,

a. find the *equilibrium price* and *quantity:*

$$\frac{-7}{100}Q + 65 = \frac{8}{100}Q + 50$$

100 = Q*
$58 = P*

b. Now, add a tax of $1.50 per unit; and find the post-policy *equilibrium price* and *quantity*.

Including the tax, the price paid by a consumer is $P_S + tax = \frac{8}{100}Q + 50 + 1.50$

$$\frac{-7}{100}Q' + 65 = \frac{8}{100}Q' + 51.50$$

Q' = 90
P' = $58.70

11. Total cost, TC = 120 + 45Q – Q^2 + $0.4Q^3$.

The requested identity equations are:
Fixed Cost (FC) **= 120**
Average Cost (AC) ≡ FC/Q **= 120/Q + 45 – Q + $0.4Q^2$**
Variable Cost (VC) **= 45Q – Q^2 + $0.4Q^3$**