KEY RECO 250 Homework #6 review for Exam #2 Fall 2009 **by Prof K**

'curve sketching' WITHOUT using excel or a graphing calculator:

1) how many wiggles does the function have? answer: the highest power – 1

2) find the vertical axis *intercept* : evaluate the function at 0; and label it (0, #). If it's not too hard evaluate the *intercept*(s) on the horizontal axis and label them (#,0).

3) identify the *critical point(s)* : take the first derivative, *f'*, set it = 0, and solve for the domain variable. **Also check if *x* = 0* when your function has more than one wiggle**. Next, evaluate the function at the *critical point*(s).By convention, label *critical points* with asterisks, e.g., *(x*, y*)*.

4) check for (local) *concavity* or *convexity*: take the second derivative, *f''*, and evaluate it at the critical points (found above). If *f''(x*)* ≥0, its "going up" around x*, so the function is *convex* at that critical point. That critical point is a local or global ***minimum***. If *f'' (x*) ≤0*, it is "going down" around x*, so the function is *concave* at that critical point. That critical point is a local or global **maximum**.

If *f''' = 0*, the function is <u>either</u> a linear function (you might have recognized that when you counted "zero wiggles," but it could also have a flat spot) <u>OR</u> *x** is an ***inflection*** point. Evaluate *f''* two more times: (i) at *x*-1* (on the left side of the critical point), and (ii) at *x*+1* (on the right side of the critical point).

(i) if *f''(x*-1)*≤0, the function's *concave* to the LEFT of the critical point; if *f''(x*-1)*≥0, its *convex* to the left of the critical point.

(ii) if *f''(x*+1)*≤0, the function's *concave* to the RIGHT of the critical point; if *f''(x*+1)*≥0, its *convex* to the right of the critical point.

NOTE: If x* isn't a nice round number to begin with, just round down to a nice round number for the left side (i), and round up to a nice round number on the right side(ii) when you re-evaluate *f''*.

1. Define each <u>concept</u> in *English* <u>and</u> provide an example (an equation or a graph).

a) the *Quotient Rule is the algorithm we use when the function f(x) is actually a ratio of two functions. What we do is to label the function on top (the numerator) as one function g(x), and take its derivative, which we label as g'. Then we label the function on the bottom (the denominator) as another function, h(x), and take its derivative, h'. We also write that function squared, $[h(x)]^2$. Then we put all the pieces back together. Problem 3 (below) is an example of where the Quotient Rule should be applied. NOTE: <u>don't</u> use the quotient rule for Problem 2!*

The Quotient Rule: when

$$y = f(x) = \frac{g(x)}{h(x)}$$

$$\frac{dy}{dx} = \frac{g'h(x) - h'g(x)}{[h(x)]^2}$$

b) the *Chain Rule is the algorithm we use when the function f(x) is actually a power function of a function, f[g(x)]. We call the power function part the 'outside function' and other function the 'inside' function, g(x). To find the derivative, we take the derivative of the outside function (and just rewrite the inside function inside again, the way it was), and multiply that by the derivative of the inside function. Problems 4 and 5 (below) are examples where the 'chain rule' should be applied.*

The Chain rule: when

$$y = f[g(x)]$$

$$\frac{dy}{dx} = \frac{df[\cdot]}{dg} \times \frac{dg(x)}{dx}$$

c) *critical value is the explanatory variable value at which the function twists or turns or reaches a maximum or minimum. We find them by taking the first derivative with respect to the explanatory variable, setting that derivative to zero, and solving for the explanatory variable. We label them x*. Maximum and minimum values are also known as a <u>relative extrema</u>. Problem 7e below is an example of a problem regarding critical points.*

d) *local maximum is a critical point (x*, f(x*)) around which the function is lower, or going down on both the left and the right. We verify if a critical point, (x*,y*) is a maximum by taking the second derivative of the original function with respect to the explanatory variable, and evaluating it at x*. If that's negative we know the function is going down around the critical point, in that case x*,y* is either a local or global maximum. Problem 7e below is an example of a problem.*

e) *First-Order Necessary Condition is the label we give to the step in our recipe for finding relative extrema where we set the first derivative with respect to the explanatory variable to zero. That step formalizes that we are looking for "the top of the hill – where the slope (the derivative!) flattens out (is zero!), or the "the bottom of the hill – where the slope (the derivative) ALSO flattens out (is zero).*

f) *Second-Order Sufficient Condition is how we label the step in the recipe for characterizing critical points as maxima, minima, or inflections points. In this step take the second derivative, f'', and evaluate it at the critical point. If f''(x*) ≥0, the function is "going up" around x*, so the function is convex at that critical point, and it is a local or global* **minimum**. *If f'' (x*) ≤0, it is "going down" around x*, so the function is concave at that critical point and it a local or global* **maximum**. *For f''(x*) = 0, see step 4 in the recap above.)*

g) *Marginal Revenue is the change in total revenue associated with a marginal change in the quantity sold. It's the first derivative of R(q) with respect to q.*

2. derivate: $y = \frac{x^2+1}{5}$ $y' = \frac{2}{5}x$

3. derivate: $y = \frac{3x}{2x+1}$ $y' = \frac{3}{(2x+1)^2}$

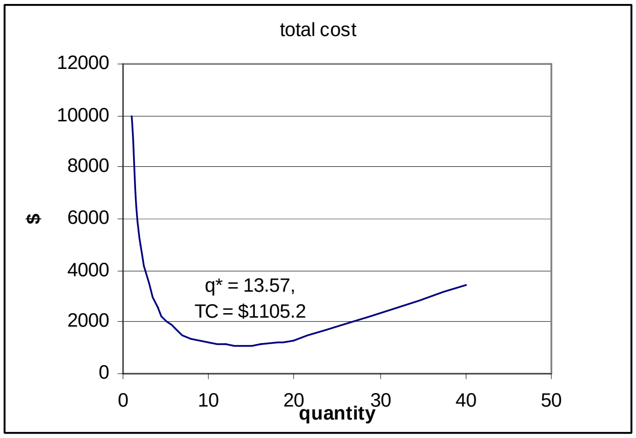
4. derivate: $y = \sqrt[3]{4x-1}$ $y' = \frac{4}{3}(4x-1)^{\frac{-2}{3}}$

5. derivate: $y = 8x^3(1-7x)^4$ $y' = 24x^2(1-7x)^4 - 28(1-7x)^3 8x^3$

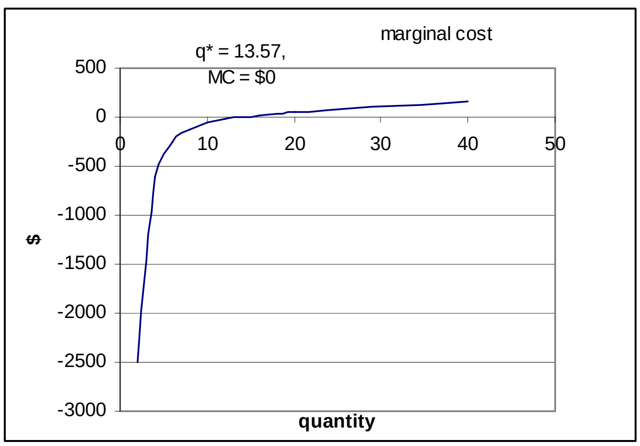
6. If *average cost* $= 2q + \dfrac{10{,}000}{q^2}$

(a) *total cost* $C(q) \equiv AC \cdot q = 2q^2 + \dfrac{10{,}000}{q}$

(b)



c) *marginal cost* $MC \equiv C'(q) \equiv \dfrac{dC}{dq} = 4q - \dfrac{10{,}000}{q^2}$



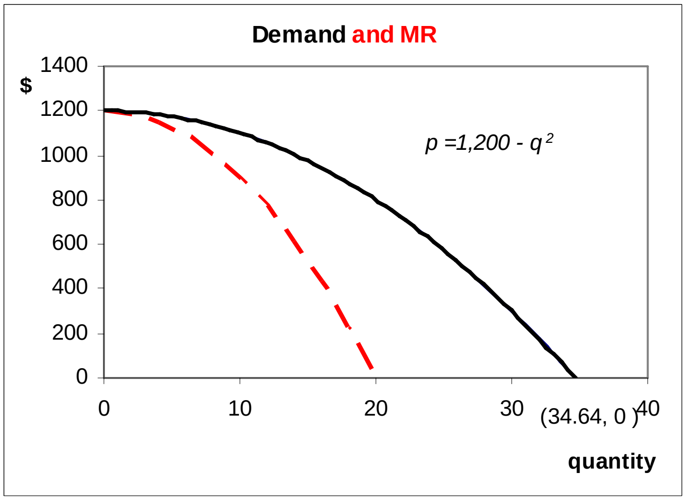
d)

7. Given *inverse demand* $p = 1,200 - q^2$

(a) total *revenue* $R \equiv p(q) \cdot q = 1,200q - q^3$

(b) *marginal revenue* $MR \equiv \frac{dR}{dq} = 1,200 - 3q^2$

(c & d)



(e) the level of q that maximizes total revenue:

By FONC : MR at $q^* = 0$ → $1,200 = 3q^2$ → $q^* = 20$

Check SOC: $\frac{dMR}{dq} = -6q \quad < 0 \qquad \forall q > 0$

because the second derivative is negative at any positive level of q, the total revenue function is 'going down everywhere' or, is *concave*, and $q^*=20$ is a *maximum* for total revenue.

8. Draw the function $y = 5x^3 - 30x^2 + 300$ without using excel or a graphing calculator.

1) it has 2 wiggles

2) *intercepts*: when $x=0,\ y = 300$

3) *critical points*: $y' = 15x^2 - 60x$

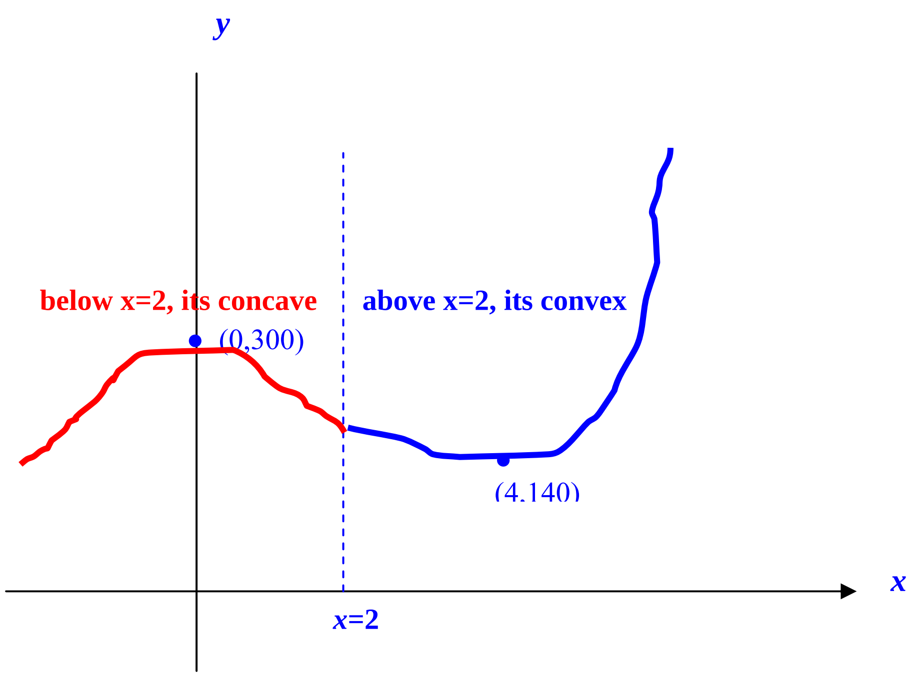
Solve $y' = 0$ and we find that the FONC holds when x^* is either *4 or 0*

Thus, $(x^*, y^*) = (4, 140)$ at one wiggle and $(0,300)$ at the other wiggle

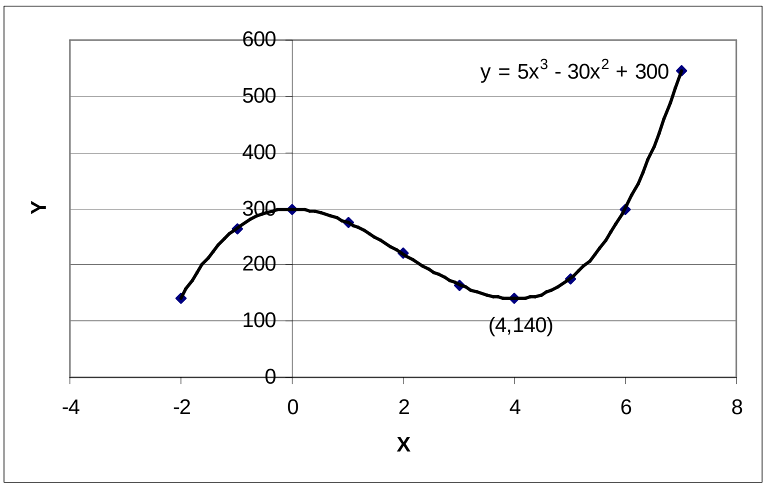
4) check *SOCs*:

(i) y'' evaluated at x*=4 is 30(4)-60 =60 which is >0, so *f(x)* is convex there; and (4, 140) is a *local minimum.*

(ii) evaluated at x*=0, y'' = -60 which is negative, or 'going down everywhere' so *f(x)* is concave there; and (0,300) is a *local maximum*. Therefore, our function looks like:



Check our work by using excel:



(And you can see why I don't put **MY** hand-drawn graphs into the answer keys!)